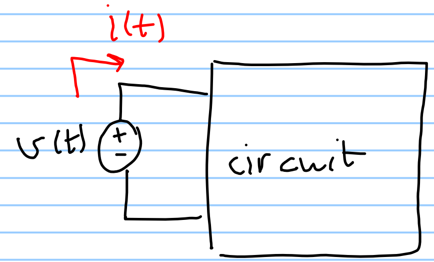
Chapter 10 Sinusoidal steady state Power calculation



↳ Instantaneous Power $p(t)$

$$p(t) = v(t)\, i(t)$$

$$= V_m \cos(\omega t + \theta_v)\, I_m \cos(\omega t + \theta_i)$$

$$= \frac{1}{2} V_m I_m \Big[\underbrace{\cos(\theta_v - \theta_i)}_{\text{constant}} + \underbrace{\cos(2\omega t + \theta_v + \theta_i)}_{\text{Twice the excitation frequency}} \Big]$$

$\rightarrow v(t) = V_m \cos(\omega t + \theta_v)$

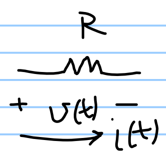
$\rightarrow i(t) = I_m \cos(\omega t + \theta_i)$

↳ Average Power : Real Power

$$P_{av} = \frac{1}{T} \int_0^T p(t)\, dt = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$\therefore \boxed{P_{av} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)} \quad \boxed{W} \text{ watt}$$

R



$\theta_v - \theta_i = \text{Zero}$

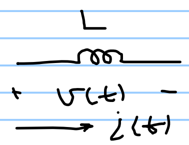
$\theta_Z = 0^\circ$

$\therefore P_{av} = \frac{1}{2} V_m I_m$

$= \frac{1}{2} \frac{V_m^2}{R}$

$= \frac{1}{2} I_m^2 R$

L



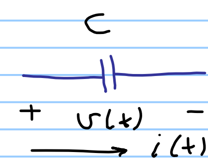
$\theta_v - \theta_i = 90^\circ$

$\theta_Z = 90^\circ$ ($j\omega L$)

$P_{av} = \text{Zero}$

$\boxed{\cos(90^\circ) = 0}$

C



$\theta_v - \theta_i = -90^\circ$

$\theta_Z = -90^\circ$ $\left(-j\frac{1}{\omega C}\right)$

$P_{av} = \text{Zero}$

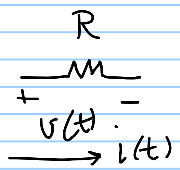
$\boxed{\cos(-90^\circ) = 0}$

Reactive Impedances absorb <u>NO</u> average power

Effective <u>OR</u> rms value   RMS :- Root Mean Square

$$V_{eff} = V_{rms} = \frac{V_m}{\sqrt{2}}, \quad I_{eff} = I_{rms} = \frac{I_m}{\sqrt{2}}$$

$$\therefore \boxed{\begin{aligned} P_{av} &= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) \\ &= V_{rms} I_{rms} \cos(\theta_v - \theta_i) \end{aligned}}$$



$P_{av} = V_{rms} I_{rms} \cos(0^\circ)$, where $\theta_v - \theta_i = 0^\circ$ R

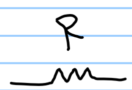
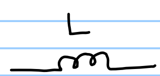
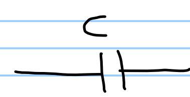
↳ $V_{rms} = R\, I_{rms}$

$$P_{av} = \frac{V_{rms}^2}{R} = I_{rms}^2 R$$

Apparent Power & Power Factor

$$P_{av} = \underbrace{V_{rms} I_{rms}}_{\substack{\text{apparent Power} \\ P_a \\ (VA)}} \underbrace{\cos(\theta_v - \theta_i)}_{\substack{\text{power factor} \\ PF}} \ , (W)$$

$$\therefore P_{av} = P_a \cdot PF$$

		
$\theta_v - \theta_i = 0$	$\theta_v - \theta_i = 90^\circ$	$\theta_v - \theta_i = -90^\circ$
$\therefore PF = 1$	$\therefore PF = 0$	$\therefore PF = 0$
unity PF		

↳ for Inductive load 

$90^\circ > \theta_v - \theta_i > 0$

$1 > PF > 0$

} lagging PF (since i lags v)

$\boxed{\cos(\alpha) = \cos(-\alpha)}$

↳ for capacitive load

$0 > \theta_v - \theta_i > -90^\circ$

$1 > PF > 0$

} leading PF (since i leads v)

Complex Power $\vec{S}$

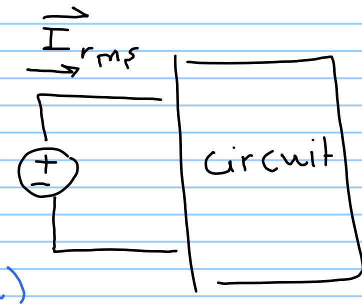
$$\vec{S} = \vec{V}_{rms} \cdot \vec{I}_{rms}^{\,*}$$

$$= V_{rms} I_{rms} \angle \theta_v - \theta_i$$

(Polar)

↓

(Rectangular)



$$= \underbrace{V_{rms} I_{rms} \cos(\theta_v - \theta_i)}_{P_{av}} + j \underbrace{V_{rms} I_{rms} \sin(\theta_v - \theta_i)}_{Q}$$

$\vec{V}_{rms} = V_{rms} \angle \theta_v$

$\vec{I}_{rms} = I_{rms} \angle \theta_i$

P_{av} average power W

Q Reactive Power VAR

$$\boxed{\vec{S} = P_{av} + jQ}$$

VA W VAR

$P_{av} = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$

$Q = V_{rms} I_{rms} \sin(\theta_v - \theta_i)$

$$P_{av} = \text{Real}[\vec{S}]$$

$$Q = \text{Imaginary}[\vec{S}]$$

$$\boxed{\vec{S} = \sqrt{P_{av}^2 + Q^2} \;\angle \tan^{-1}\frac{Q}{P_{av}}} \quad * \quad \boxed{\vec{S} = V_{rms} I_{rms} \angle \theta_v - \theta_i}$$

$$\boxed{\vec{S} = P_{av} + jQ}$$

R

$\theta_v - \theta_i = 0$

$P_{av} = V_{rms} I_{rms}$

$Q_R = \text{Zero}$

L $(j\omega L)$

$\theta_v - \theta_i = 90^\circ$

$P_{av_L} = \text{Zero}$

$$\boxed{Q_L = V_{rms} I_{rms}}$$

$\therefore V_{rms}$ (not $\vec{V}_{rms}$)

$V_{rms} = (\omega L) I_{rms}$

$$\boxed{Q_L = \omega L I_{rms}^2 = \frac{V_{rms}^2}{\omega L}}$$

C $\left(-j\frac{1}{\omega C}\right)$

$\theta_v - \theta_i = -90^\circ$

$P_{av_C} = \text{Zero}$

$\sin(-90^\circ) = -1$

$$\boxed{Q_C = -V_{rms} I_{rms}}$$

$I_{rms} = \omega C V_{rms}$

$$\boxed{Q_C = -\frac{I_{rms}^2}{\omega C} = -\omega C V_{rms}^2}$$

R	$L\ (j\omega L)$	$C\ \left(-j\frac{1}{\omega C}\right)$
$\theta_v - \theta_i = 0$	$\theta_v - \theta_i = 90^\circ$	$\theta_v - \theta_i = -90^\circ$
$P_{av} = V_{rms} I_{rms}$	$P_{av_L} = \text{Zero}$	$P_{av_C} = \text{Zero}$
$Q_R = \text{Zero}$	$\boxed{Q_L = V_{rms} I_{rms}}$	$\boxed{Q_C = -V_{rms} I_{rms}}$
	∴ V_{rms} (not $\vec{V}_{rms}$)	
	$V_{rms} = (\omega L) I_{rms}$	$I_{rms} = \omega C V_{rms}$
	$\boxed{Q_L = \underbrace{(\omega L)}_{X_L} I_{rms}^2 = \frac{V_{rms}^2}{\omega L}}$	$\boxed{Q_C = \underbrace{-\frac{1}{\omega C}}_{X_C} I_{rms}^2 = -\omega C V_{rms}^2}$

$$Q_L = (X_L) I_{rms}^2 \longleftrightarrow Q_C = X_C I_{rms}^2$$

chapter (9)

$$Z = R + j\underline{\underline{X}}$$

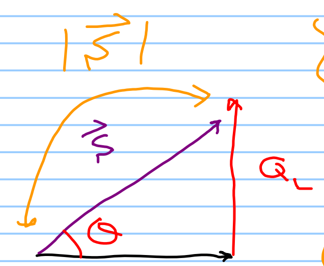
impedance — reactance

↳ for $j\omega L$ (inductor) $X_L = \omega L$

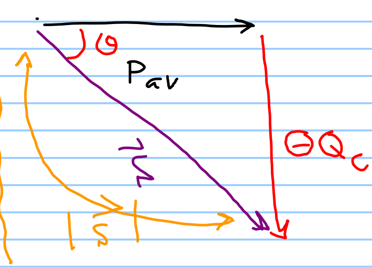
↳ for $-j\frac{1}{\omega C}$ (capacitor) $X_C = \frac{-1}{\omega C}$

Power Triangle

P_{av} $\quad$ Q → L: +ve, C: −ve $\quad$ $\vec{S}$



$\vec{S} = P_{av} + jQ_L$



−ve

$\vec{S} = P_{av} + jQ_C$

$\theta = \theta_v - \theta_i$

∴ $PF = \cos\theta = \frac{P_{av}}{|\vec{S}|}$

& $\sin\theta = \frac{Q_L}{|\vec{S}|}$

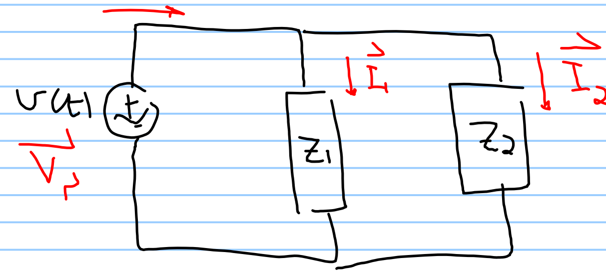
↳ $\tan\theta = \frac{Q}{P_{av}} \Rightarrow \boxed{\theta = \tan^{-1}\frac{Q}{P_{av}}}$

$$\underline{\underline{P_a}} = |\vec{S}| = \sqrt{P_{av}^2 + Q^2}$$

↳ apparent power

Conservation of AC Power

The complex, Real, and Reactive powers of the sources equal the respective sum of the complex, real, and reactive powers of the individual Loads.



$$\vec{S}_{source} = \vec{V}_s \cdot \vec{I}_s^{\,*}$$
$$= \vec{V}_s \cdot (\vec{I}_1^{\,*} + \vec{I}_2^{\,*})$$
$$= \vec{V}_s \vec{I}_1^{\,*} + \vec{V}_s \cdot \vec{I}_2^{\,*}$$
$$= \vec{S}_1 + \vec{S}_2$$

$\vec{V}_s$, $\vec{I}_s$, $\vec{I}_1$ & $\vec{I}_2$ are phasors

→ The same results can be obtained for a series connection →

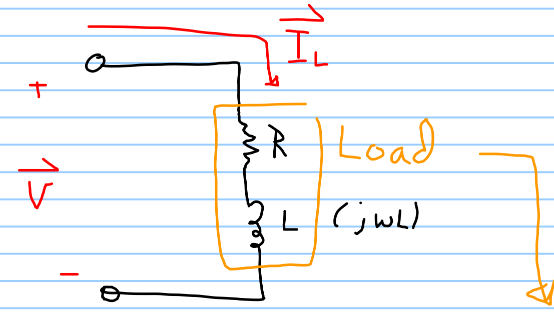
→ $P_{av_T} = P_{av_1} + P_{av_2} + \cdots + P_{av\,n}$

→ $Q_T = Q_1 + Q_2 + \cdots + Q_n$

$$\vec{S}_T = P_{av_T} + j\,Q_T = \vec{S}_1 + \vec{S}_2 + \cdots + \vec{S}_n$$

Power Factor Correction

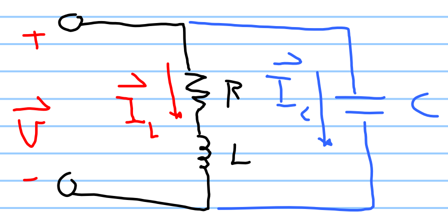
↱ The power factor correction is the process of increasing the power factor without altering the voltage OR current to the original load.



↱ Power factor correction is necessary for economic reason.

$$PF = \cos(\theta_v - \theta_i)$$

↱ for $\underline{\underline{R}}$ $PF = 1$ & $Q_R = 0$



↱ To improve the PF, we must decrease the reactive power!

↱ for inductive circuit, we add a capacitor in parallel to the load

$$Q_c = Q_{Final} - Q_{init.}$$

$$\boxed{Q_c = \frac{V_{rms}^2}{X_c} = -wc\,V_{rms}^2}$$

$$\boxed{C = \frac{-Q_c}{w V_{rms}^2}}$$